

# Possible Dark States Induced by a Surface Wave Along a Vacuum-Matter Boundary

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**Abstract** Possible dark states could be induced after derivations of the entrainment of matter induced by a surface wave propagating along the flexible vacuum-matter boundary by considering the nonlinear coupling between the interface and the rarefaction effect. The non-relativistic limit of the relativistic Navier-Stokes equations was considered and analytically solved by a perturbation approach. The critical reflux values associated with the product of the second-order body forcing and the Reynolds number (representing the viscous dissipations) decrease as the Knudsen number (representing the rarefaction measure) increases from zero to 0.1. We obtained the critical bounds for possible dark states corresponding to specific Reynolds numbers (ratio of wave inertia and viscous dissipation effects) and wave numbers which might be linked to the dissipative evolution of certain large-scale structure during the relativistic heavy-ion collisions.

**Keywords** Dissipative soliton · Dark matter · Casimir effect · Slip

## 1 Introduction

Recently a new state of matter has been created in Au+Au collisions at RHIC, and surprisingly it was found to flow as a perfect fluid [1]. The kinematic shear viscosity of this nearly perfect fluid has been determined and found to be rather small compared to conventional low-temperature fluids. Relevant researches in heavy-ion physics has focused on constraining the transport properties of hot and dense nuclear matter using experimental data from RHIC and studying relativistic hydrodynamics for viscous fluids in order to describe the expansion of the fireballs created in relativistic heavy-ion collisions.

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Meanwhile the mean cosmic density of dark matter (plus baryons) is now pinned down to be only ca. 30% of the so-called critical density corresponding to a ‘flat’-Universe. However, other recent evidencemicrowave background anisotropies, complemented by data on distant supernovae reveals that our Universe actually is ‘flat’, but that its dominant ingredient (ca. 70% of the total mass energy) is something quite unexpected: ‘dark energy’ pervading all space, with negative pressure. We do know that this material is very dark and that it dominates the internal kinematics, clustering properties and motions of galactic systems. Dark matter is commonly associated to weakly interacting particles (WIMPs), and can be described as a fluid with vanishing pressure. It plays a crucial role in the formation and evolution of structure in the universe and it is unlikely that galaxies could have formed without its presence [2]. Analysis of cosmological mixed dark matter models in spatially flat Friedmann Universe with zero  $\Lambda$  term have been presented before. A large majority of dark energy models describes dark energy in terms of the equation of state (EOS)  $p_d = \omega \rho_d$  (cf. [3, 4]), where  $\omega$  is the parameter of the EOS, while  $p_d$  and  $\rho_d$  denote the pressure and the energy density of dark energy, respectively. The value  $\omega = -1$  is characteristic of the cosmological constant, while the dynamical models of dark energy generally have  $\omega \geq -1$ . The case of the growing cosmological term  $\Lambda$  and its implications for the asymptotic expansion of the universe and the destiny of the bound systems have been studied in [4] using above system of equations. Their results showed that even for very slow growth of  $\Lambda$  (which satisfies all the conditions on the variation of  $G_N$ ), in the distant future the gravitationally bound systems become unbound, while the non-gravitationally bound systems remain bound.

Influential only over the largest of scales-the cosmological horizon-is the outermost species of invisible matter: the vacuum energy (also known by such names as dark energy, quintessence,  $x$ -matter, the zero-point field, and the cosmological constant  $\Lambda$ ) (cf. [5, 6]). If there is no exchange of energy between vacuum and matter components, the requirement of general covariance implies the time dependence of the gravitational constant  $G$ . Thus, it is interesting to look at the interacting behavior between the vacuum (energy) and the matter from the macroscopic point of view. One related issue, say, is about the dissipative matter of the flat Universe immersed in vacua [7] and the other one is the macroscopic Casimir effect with the deformed boundaries [8].

Theoretical (using the Boltzmann equation) and experimental studies of interphase non-local transport phenomena which appear as a result of a different type of nonequilibrium representing propagation of a surface elastic wave have been performed since late 1980s (cf. [9–11]). These are relevant to rarefied gases (RG) flowing along deformable elastic slabs with the dominated parameter being the Knudsen number ( $\text{Kn} = \text{mean-free-path}/L_d$ , mean-free-path (mfp) is the mean free path of the gas,  $L_d$  is proportional to the distance between two slabs) [12–14]. The role of the Knudsen number is similar to that of the Navier slip parameter  $N_s$  (cf. [15]); here,  $N_s = \mu S/d$  is the dimensionless Navier slip parameter;  $S$  is a proportionality constant as  $u_s = S\tau$ ,  $\tau$ : the shear stress of the bulk velocity;  $u_s$ : the dimensional slip velocity; for a no-slip case,  $S = 0$ , but for a no-stress condition.  $S = \infty$ ,  $\mu$  is the fluid viscosity,  $d$  is one half of the distance between upper and lower slabs).

Note that, there are some models, like the MIT bag model and its descendants, where matter is in a bag, in which there is no vacuum and the vacuum is outside. In this particular model one might speak of a clear *vacuum-matter boundary*. Here, borrowing the idea of the MIT bag model, the transport driven by the wavy elastic vacuum-matter boundary will be presented. The flat-Universe is presumed and the corresponding matter is immersed in vacua with the interface being flat-plane like. We adopt the macroscopic or hydrodynamical approach and simplify the original system of equations (related to the momentum and mass

transport) to one single higher-order quasi-linear partial differential equation in terms of the unknown stream function. We then introduce the perturbation technique so that we can solve the related boundary value problem approximately. To consider the originally quiescent gas for simplicity, due to the difficulty in solving a fourth-order quasi-linear complex ordinary differential equation (when the wavy boundary condition are imposed), we can finally get an analytically perturbed solution and calculate those physical quantities we have interests, like, time-averaged transport or entrainment, perturbed velocity functions, critical unit body forcing corresponding to the possible dark states. These results might be closely linked to the vacuum-matter interactions (say, macroscopic Casimir effects) and the evolution of the Universe (as mentioned above: the critical density [2]). Our results also show that for certain time-averaged evolution of the matter (the maximum speed of the matter (gas) appears at the center-line) there might be existence of negative-pressure states.

## 2 Formulations

The matter is presumed to be a fluid associated with a shear viscosity but no bulk viscosity and no heat conduction here (the geometrized units are adopted so that  $G = c = 1$  and the Einstein’s field equations:  $G_{\mu\nu} = 8\pi T_{\mu\nu}$ ). The stress tensor (for this fluid) is [16, 17]

$$T_{\mu\nu} = (\rho_0 + \rho_0 e_0 + P)u_\mu u_\nu + P g_{\mu\nu} - 2\eta\sigma_{\mu\nu}.$$

Here,  $\rho_0$ ,  $e_0$ ,  $P$ , and  $u_\mu$  are the rest-mass density, specific internal energy, pressure, and the fluid 4-velocity, respectively.  $\eta$  is the coefficient of viscosity and is related to the kinematic viscosity  $\nu$  by  $\eta = \rho_0\nu$ .  $\sigma_{\mu\nu}$  is the shear tensor (the detailed expression could be traced in [16, 17]). In general a  $\Gamma$ -law equation of state  $P = (\Gamma - 1)\rho_0 e_0$  could be presumed. Thus, we have basic fluid variables

$$\rho_* = \rho_0 \alpha u^0 e^{6\phi}, \quad e_* = (\rho_0 e_0)^{1/\Gamma} \alpha u^0 e^{6\phi}, \quad \tilde{S}_k = \rho_* h u_k,$$

where  $\phi$  is the conformal exponent and  $h = 1 + e_0 + P/\rho_0$  is the specific enthalpy ( $\alpha$  is the elapse or proper time elapsed in moving between the neighbouring spatial hypersurfaces). The conservation of stress-energy [16, 17] gives  $T_{;\nu}^{\mu\nu} = 0$  and the law of baryon number conservation  $\nabla_\mu(\rho_0 u^\mu) = 0$  gives the relativistic continuity, energy, and Navier-Stokes equations

$$\partial_t \rho_* + \partial_i(\rho_* v^i) = 0, \tag{1}$$

$$\partial e_* + \partial_i(e_* v^i) = \frac{2}{\Gamma} \alpha e^{6\phi} \eta (\rho_0 e_0)^{(1-\Gamma)/\Gamma} \sigma^{\alpha\beta} \sigma_{\alpha\beta}, \tag{2}$$

$$\partial_i \tilde{S}_k + \partial_i(\tilde{S}_k v^i) = -\alpha e^{6\phi} P_{,k} + 2(\alpha e^{6\phi} \eta \sigma_k^\mu)_{,\mu} + \alpha e^{6\phi} g_k^{\alpha\beta} \left( \eta \sigma_{\alpha\beta} - \frac{1}{2} \rho_0 h u_\alpha u_\beta \right), \tag{3}$$

where  $v^i = u^i/u_0$  is the 3-velocity. The quantity  $u^0$  is determined by the normalization condition  $u^\nu u_\nu = 1$ , which has

$$w^2 = \rho_*^2 + e^{-4\phi} \tilde{\gamma}^{ij} \tilde{S}_i \tilde{S}_j \left[ 1 + \frac{\Gamma e_*^\Gamma}{\rho_* (w e^{6\phi} / \rho_*)^{\Gamma-1}} \right]^{-2},$$

with  $w = \rho_* \alpha u^0$  ( $\gamma_{ij}$  is the spatial or 3-metric). We remind the readers that the stress tensor  $T^{\mu\nu}$  generates the source terms in the field evolution equations:

$$\begin{aligned}\rho &= h w e^{-6\phi} - P - \frac{2\eta}{\alpha^2} (\sigma_{ii} - 2\sigma_{ii}\beta^i + \sigma_{ij}\beta^i\beta^j), \\ S_i &= e^{-6\phi} \tilde{S}_i - \frac{2\eta}{\alpha} (\sigma_{ii} - \sigma_{ij}\beta^j), \\ S_{ij} &= \frac{e^{-6\phi}}{wh} \tilde{S}_i \tilde{S}_j + P\gamma_{ij} - 2\eta\sigma_{ij},\end{aligned}$$

where  $\beta^i$  is the shift (for gauge conditions) or displacement in spatial coordinates in moving between the neighbouring spatial hypersurfaces.

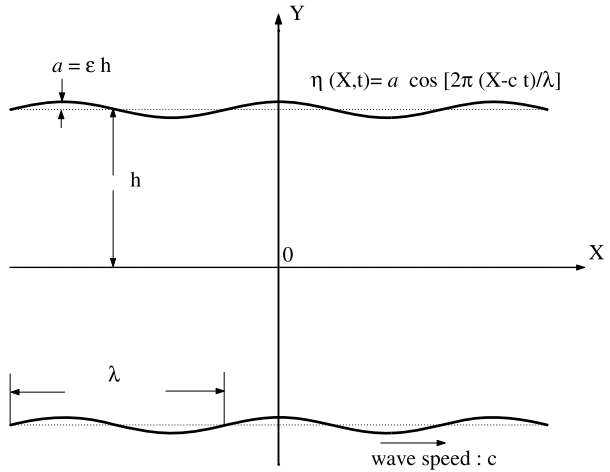
Equations (1)–(3) are too difficult to be solved analytically or even by a perturbation approach. In this work we only consider the nonrelativistic limit of above equations. Meanwhile the vacuum is presumed to be incompressible (cf., e.g., [18], the speed of sound propagating in this ‘vacuum’ is formally very large (or infinite) rather than zero as in the empty vacuum; this implies the incompressible vacuum).

Note that the first theories of relativistic dissipative fluid dynamics are due to Eckart [19] and to Landau and Lifshitz [20]. The difference in formal appearance stems from different choices for the definition of the hydrodynamical four-velocity. These conventional theories of dissipative fluid dynamics are based on the assumption that the entropy four-current contains terms up to linear order in dissipative quantities and hence they are referred to as *first order* theories of dissipative fluids. The resulting equations for the dissipative fluxes are linearly related to the thermodynamic forces, and the resulting equations of motion are parabolic in structure, from which we get the Fourier-Navier-Stokes equations. They have the undesirable feature that *causality* may not be satisfied. That is, they may propagate viscous and thermal signals with speeds exceeding that of light. Extended theories of dissipative fluids due to Grad [21], Müller [22], and Israel and Stewart [23, 24] were introduced to remedy some of these undesirable features. These causal theories are based on the assumption that the entropy four-current should include terms quadratic in the dissipative fluxes and hence they are referred to as *second order* theories of dissipative fluids. The resulting equations for the dissipative fluxes are hyperbolic and they lead to causal propagation of signals [25, 26]. A qualitative study of relativistic dissipative fluids for applications to relativistic heavy ions collisions has been done using these first order theories. The application of second order theories to nuclear collisions has just begun [27].

The flat-plane boundaries of this matter-region or the vacuum-matter boundaries are rather flexible and presumed to be elastic, on which are imposed traveling sinusoidal waves of small amplitude  $a$  (possibly due to vacuum fluctuations). The vertical displacements of the upper and lower interfaces ( $y = h$  and  $-h$ ) are thus presumed to be  $\eta$  and  $-\eta$ , respectively, where  $\eta = a \cos[2\pi(x - ct)/\lambda]$ ,  $\lambda$  is the wave length, and  $c$  the wave speed.  $x$  and  $y$  are Cartesian coordinates, with  $x$  measured in the direction of wave propagation and  $y$  measured in the direction normal to the mean position of the vacuum-matter interfaces. The schematic plot of above features is shown in Fig. 1.

It would be convenient to simplify these equations by introducing dimensionless variables. We have a characteristic velocity  $c$  and three characteristic lengths  $a$ ,  $\lambda$ , and  $h$ . The following variables based on  $c$  and  $h$  could thus be introduced:  $x' = x/h$ ,  $y' = y/h$ ,  $u' = u/c$ ,  $v' = v/c$ ,  $\eta' = \eta/h$ ,  $\psi' = \psi/(ch)$ ,  $t' = ct/h$ ,  $p' = p/(\rho c^2)$ , where  $\psi$  is the dimensional stream function,  $u$  and  $v$  are the velocities along the  $x$ - and  $y$ -directions;  $\rho$  is the density,  $p$  (its gradient) is related to the (unit) body forcing. The primes could be dropped in

**Fig. 1** Schematic diagram of the deformable motion of the vacua-matter boundary



the following. The amplitude ratio  $\epsilon$ , the wave number  $\alpha$ , and the Reynolds number (ratio of wave inertia and viscous dissipation effects)  $Re$  are defined by

$$\epsilon = \frac{a}{h}, \quad \alpha = \frac{2\pi h}{\lambda}, \quad Re = \frac{c h}{\nu}.$$

We shall seek a solution in the form of a series in the parameter  $\epsilon$ :  $\psi = \psi_0 + \epsilon \psi_1 + \epsilon^2 \psi_2 + \dots$ ,  $\partial p / \partial x = (\partial p / \partial x)_0 + \epsilon (\partial p / \partial x)_1 + \epsilon^2 (\partial p / \partial x)_2 + \dots$ , with  $u = \partial \psi / \partial y$ ,  $v = -\partial \psi / \partial x$ . The two-dimensional ( $x$ - and  $y$ -) momentum equations and the equation of continuity could be in terms of the stream function  $\psi$  if the  $p$ -term (the specific body force density, assumed to be conservative and hence expressed as the gradient of a time-independent potential energy function) is eliminated. The final governing equation is

$$\frac{\partial}{\partial t} \nabla^2 \psi + \psi_y \nabla^2 \psi_x - \psi_x \nabla^2 \psi_y = \frac{1}{Re} \nabla^4 \psi, \quad \nabla^2 \equiv \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}, \quad (4)$$

and subscripts indicate the partial differentiation. Thus, we have

$$\frac{\partial}{\partial t} \nabla^2 \psi_0 + \psi_{0y} \nabla^2 \psi_{0x} - \psi_{0x} \nabla^2 \psi_{0y} = \frac{1}{Re} \nabla^4 \psi_0, \quad (5)$$

$$\frac{\partial}{\partial t} \nabla^2 \psi_1 + \psi_{0y} \nabla^2 \psi_{1x} + \psi_{1y} \nabla^2 \psi_{0x} - \psi_{0x} \nabla^2 \psi_{1y} - \psi_{1x} \nabla^2 \psi_{0y} = \frac{1}{Re} \nabla^4 \psi_1, \quad (6)$$

$$\begin{aligned} \frac{\partial}{\partial t} \nabla^2 \psi_2 + \psi_{0y} \nabla^2 \psi_{2x} + \psi_{1y} \nabla^2 \psi_{1x} + \psi_{2y} \nabla^2 \psi_{0x} \\ - \psi_{0x} \nabla^2 \psi_{2y} - \psi_{1x} \nabla^2 \psi_{1y} - \psi_{2x} \nabla^2 \psi_{0y} = \frac{1}{Re} \nabla^4 \psi_2, \end{aligned} \quad (7)$$

and other higher order terms. The (matter) gas is subjected to boundary conditions imposed by the symmetric motion of the vacuum-matter interfaces and the non-zero slip velocity:  $u = \mp Kn \, du/dy$  (cf. [12–14]),  $v = \pm \partial \eta / \partial t$  at  $y = \pm(1 + \eta)$ , here  $Kn = mfp/(2h)$ . The

boundary conditions may be expanded in powers of  $\eta$  and then  $\epsilon$ :

$$\begin{aligned} &\psi_{0y}|_1 + \epsilon[\cos \alpha(x-t)\psi_{0yy}|_1 + \psi_{1y}|_1] + \epsilon^2 \left[ \frac{\psi_{0yyy}|_1}{2} \cos^2 \alpha(x-t) + \psi_{2y}|_1 \right. \\ &\quad \left. + \cos \alpha(x-t)\psi_{1yy}|_1 \right] + \dots \\ &= -\text{Kn} \left\{ \psi_{0yy}|_1 + \epsilon[\cos \alpha(x-t)\psi_{0yyy}|_1 + \psi_{1yy}|_1] \right. \\ &\quad \left. + \epsilon^2 \left[ \frac{\psi_{0yyyy}|_1}{2} \cos^2 \alpha(x-t) + \cos \alpha(x-t)\psi_{1yyy}|_1 + \psi_{2yy}|_1 \right] + \dots \right\}, \end{aligned} \tag{8}$$

$$\begin{aligned} &\psi_{0x}|_1 + \epsilon[\cos \alpha(x-t)\psi_{0xy}|_1 + \psi_{1x}|_1] + \epsilon^2 \left[ \frac{\psi_{0xyy}|_1}{2} \cos^2 \alpha(x-t) \right. \\ &\quad \left. + \cos \alpha(x-t)\psi_{1xy}|_1 + \psi_{2x}|_1 \right] + \dots = -\epsilon \alpha \sin \alpha(x-t). \end{aligned} \tag{9}$$

Equations above, together with the condition of symmetry and a uniform  $(\partial p/\partial x)_0$ , yield:

$$\psi_0 = K_0 \left[ (1 + 2\text{Kn})y - \frac{y^3}{3} \right], \quad K_0 = \frac{Re}{2} \left( -\frac{\partial p}{\partial x} \right)_0, \tag{10}$$

$$\psi_1 = \frac{1}{2} \{ \phi(y)e^{i\alpha(x-t)} + \phi^*(y)e^{-i\alpha(x-t)} \}, \tag{11}$$

where the asterisk denotes the complex conjugate. A substitution of  $\psi_1$  into (6) yields

$$\left\{ \frac{d^2}{dy^2} - \alpha^2 + i\alpha Re[1 - K_0(1 - y^2 + 2\text{Kn})] \right\} \left( \frac{d^2}{dy^2} - \alpha^2 \right) \phi - 2i\alpha K_0 Re \phi = 0$$

or if originally the (matter) gas is quiescent:  $K_0 = 0$  (this corresponds to a free (vacuum) pumping case)

$$\left( \frac{d^2}{dy^2} - \alpha^2 \right) \left( \frac{d^2}{dy^2} - \bar{\alpha}^2 \right) \phi = 0, \quad \bar{\alpha}^2 = \alpha^2 - i\alpha Re. \tag{12}$$

The boundary conditions are

$$\phi_y(\pm 1) \pm \phi_{yy}(\pm 1)\text{Kn} = 2K_0(1 \pm \text{Kn}) = 0, \quad \phi(\pm 1) = \pm 1. \tag{13}$$

Similarly, with

$$\psi_2 = \frac{1}{2} \{ D(y) + E(y)e^{i2\alpha(x-t)} + E^*(y)e^{-i2\alpha(x-t)} \}, \tag{14}$$

we have

$$D_{yyyy} = -\frac{i\alpha Re}{2} (\phi\phi_{yy}^* - \phi^*\phi_{yy}), \tag{15}$$

$$\left[ \frac{d^2}{dy^2} - (4\alpha^2 - 2i\alpha Re) \right] \left( \frac{d^2}{dy^2} - 4\alpha^2 \right) E - i2\alpha Re K_0 (1 - y^2 + 2Kn) \\ \times \left( \frac{d^2}{dy^2} - 4\alpha^2 \right) E + i4\alpha K_0 Re E + \frac{i\alpha Re}{2} (\phi_y \phi_{yy} - \phi \phi_{yyy}) = 0; \tag{16}$$

and the boundary conditions

$$D_y(\pm 1) + \frac{1}{2} [\phi_{yy}(\pm 1) + \phi_{yy}^*(\pm 1)] - 2K_0 \\ = \mp Kn \left\{ \frac{1}{2} [\phi_{yyy}(\pm 1) + \phi_{yyy}^*(\pm 1)] + D_{yy}(\pm 1) \right\}, \tag{17}$$

$$E_y(\pm 1) + \frac{1}{2} \phi_{yy}(\pm 1) - \frac{K_0}{2} = \mp Kn \left[ \frac{1}{2} \phi_{yyy}(\pm 1) + E_{yy}(\pm 1) \right], \tag{18}$$

$$E(\pm 1) + \frac{1}{4} \phi_y(\pm 1) = 0 \tag{19}$$

where  $K_0$  is zero in (16)–(18). After lengthy algebraic manipulations, we obtain  $\phi = c_0 e^{\alpha y} + c_1 e^{-\alpha y} + c_2 e^{\bar{\alpha} y} + c_3 e^{-\bar{\alpha} y}$ , where  $c_0 = (A + A_0)/Det$ ,  $c_1 = -(B + B_0)/Det$ ,  $c_2 = (C + C_0)/Det$ ,  $c_3 = -(T + T_0)/Det$ ;  $Det = Ae^\alpha - Be^{-\alpha} + Ce^{\bar{\alpha}} - Te^{-\bar{\alpha}}$ ,

$$A = e^\alpha \bar{\alpha}^2 (r^2 e^{-2\bar{\alpha}} - s^2 e^{2\bar{\alpha}}) - 2\alpha \bar{\alpha} e^{-\alpha} w + \alpha \bar{\alpha} e^\alpha z (e^{-2\bar{\alpha}} r + e^{2\bar{\alpha}} s), \\ A_0 = e^{-\alpha} \bar{\alpha}^2 (r^2 e^{-2\bar{\alpha}} - s^2 e^{2\bar{\alpha}}) + 2\alpha \bar{\alpha} e^\alpha z - \alpha \bar{\alpha} e^{-\alpha} w (e^{2\bar{\alpha}} s + e^{-2\bar{\alpha}} r), \\ B = e^{-\alpha} \bar{\alpha}^2 (r^2 e^{-2\bar{\alpha}} - s^2 e^{2\bar{\alpha}}) + 2\alpha \bar{\alpha} e^\alpha z - \alpha \bar{\alpha} e^{-\alpha} w (e^{-2\bar{\alpha}} r + e^{2\bar{\alpha}} s), \\ B_0 = e^\alpha \bar{\alpha}^2 (r^2 e^{-2\bar{\alpha}} - s^2 e^{2\bar{\alpha}}) - 2\alpha \bar{\alpha} e^{-\alpha} w + \alpha \bar{\alpha} e^\alpha z (e^{-2\bar{\alpha}} r + e^{2\bar{\alpha}} s), \\ C = e^{-\alpha} \alpha \bar{\alpha} (w s e^{\bar{\alpha}-\alpha} - r z e^{\alpha-\bar{\alpha}}) - \alpha e^{2\alpha+\bar{\alpha}} z (\alpha z - \bar{\alpha} s) + \alpha e^{-\alpha} w (\alpha e^{\bar{\alpha}-\alpha} w - \bar{\alpha} e^{\alpha-\bar{\alpha}} r), \\ C_0 = e^\alpha \alpha \bar{\alpha} (w s e^{\bar{\alpha}-\alpha} - r z e^{\alpha-\bar{\alpha}}) - \alpha z (z \alpha e^{2\alpha-\bar{\alpha}} - \bar{\alpha} e^{\bar{\alpha}} s) + \alpha w (\alpha e^{-(\bar{\alpha}+2\alpha)} w - \bar{\alpha} e^{-(2\alpha+\bar{\alpha})} r), \\ T = e^{-\alpha} \alpha \bar{\alpha} (z s e^{\bar{\alpha}+\alpha} - r w e^{-(\alpha+\bar{\alpha})}) - \alpha \bar{\alpha} (e^{2\alpha-\bar{\alpha}} z r - e^{\bar{\alpha}} w s) + \alpha^2 e^{-\alpha} (-e^{2\alpha} z^2 + e^{-2\alpha} w^2), \\ T_0 = e^\alpha \alpha \bar{\alpha} (z s e^{\bar{\alpha}+\alpha} - r w e^{-(\alpha+\bar{\alpha})}) - \alpha \bar{\alpha} (e^{-\bar{\alpha}} z r - e^{\bar{\alpha}-2\alpha} w s) + \alpha^2 e^\alpha (-e^{2\alpha} z^2 + e^{-2\alpha} w^2),$$

with  $r = (1 - \bar{\alpha}Kn)$ ,  $s = (1 + \bar{\alpha}Kn)$ ,  $w = (1 - \alpha Kn)$ ,  $z = (1 + \alpha Kn)$ .

To obtain a simple solution which relates to the mean transport so long as only terms of  $O(\epsilon^2)$  are concerned, we see that if every term in the x-momentum equation is averaged over an interval of time equal to the period of oscillation, we obtain for our solution as given by above equations the time-averaged (unit) body forcing

$$\overline{\frac{\partial p}{\partial x}} = \epsilon^2 \overline{\left( \frac{\partial p}{\partial x} \right)}_2 = \epsilon^2 \left[ \frac{D_{yyy}}{2Re} + \frac{iRe}{4} (\phi \phi_{yy}^* - \phi^* \phi_{yy}) \right] + O(\epsilon^3) = \epsilon^2 \frac{\Pi_0}{Re} + O(\epsilon^3), \tag{20}$$

where  $\Pi_0$  is the integration constant for the integration of (15) and could be fixed indirectly in the coming equation below. Now, from (17), we have

$$D_y(\pm 1) \pm Kn D_{yy}(\pm 1) = -\frac{1}{2} [\phi_{yy}(\pm 1) + \phi_{yy}^*(\pm 1)] \mp Kn \left\{ \frac{1}{2} [\phi_{yyy}(\pm 1) + \phi_{yyy}^*(\pm 1)] \right\}, \tag{21}$$

where  $D_y(y) = \Pi_0 y^2 + a_1 y + a_2 + \mathcal{C}(y)$ , and together from (15), we obtain

$$\begin{aligned} \mathcal{C}(y) = & \frac{\alpha^2 Re^2}{2} \left[ \frac{c_0 c_2^*}{g_1^2} e^{(\alpha + \bar{\alpha}^*)y} + \frac{c_0^* c_2}{g_2^2} e^{(\alpha + \bar{\alpha})y} + \frac{c_0 c_3^*}{g_3^2} e^{(\alpha - \bar{\alpha}^*)y} + \frac{c_0^* c_3}{g_4^2} e^{(\alpha - \bar{\alpha})y} \right. \\ & + \frac{c_1 c_2^*}{g_3^2} e^{(\bar{\alpha}^* - \alpha)y} + \frac{c_1^* c_2}{g_4^2} e^{(\bar{\alpha} - \alpha)y} + \frac{c_1 c_3^*}{g_1^2} e^{-(\bar{\alpha}^* + \alpha)y} + \frac{c_1^* c_3}{g_2^2} e^{-(\bar{\alpha} + \alpha)y} \\ & \left. + \frac{c_2 c_3^*}{g_5^2} e^{(\bar{\alpha} - \bar{\alpha}^*)y} + \frac{c_2^* c_3}{g_5^2} e^{(\bar{\alpha}^* - \bar{\alpha})y} + 2 \frac{c_2 c_2^*}{g_6^2} e^{(\bar{\alpha}^* + \bar{\alpha})y} + 2 \frac{c_3 c_3^*}{g_6^2} e^{-(\bar{\alpha}^* + \bar{\alpha})y} \right], \end{aligned} \tag{22}$$

with  $g_1 = \alpha + \bar{\alpha}^*$ ,  $g_2 = \alpha + \bar{\alpha}$ ,  $g_3 = \alpha - \bar{\alpha}^*$ ,  $g_4 = \alpha - \bar{\alpha}$ ,  $g_5 = \bar{\alpha} - \bar{\alpha}^*$ ,  $g_6 = \bar{\alpha} + \bar{\alpha}^*$ . In realistic applications we must determine  $\Pi_0$  from considerations of conditions at the ends of the matter-region.  $a_1$  equals to zero because of the symmetry of boundary conditions.

Once  $\Pi_0$  is specified, our solution for the mean speed ( $u$  averaged over time) of matter-flow is

$$U = \epsilon^2 \frac{D_y}{2} = \frac{\epsilon^2}{2} \{ \mathcal{C}(y) - \mathcal{C}(1) + R_0 - \text{Kn} C_y(1) + \Pi_0 [y^2 - (1 + 2\text{Kn})] \} \tag{23}$$

where  $R_0 = -\{[\phi_{yy}(1) + \phi_{yy}^*(1)] - \text{Kn}[\phi_{yyy}(1) + \phi_{yyy}^*(1)]\}/2$ , which has a numerical value about 3 for a wide range of  $\alpha$  and  $Re$  (playing the role of viscous dissipations) when  $\text{Kn} = 0$ . To illustrate our results clearly, we adopt  $U(Y) \equiv u(y)$  for the time-averaged results with  $y \equiv Y$  in the following.

### 3 Results and Discussion

We check our approach firstly by examining  $R_0$  with that of no-slip ( $\text{Kn} = 0$ ) approach. This can be done easily once we consider terms of  $D_y(y)$  and  $\mathcal{C}(y)$  because to evaluate  $R_0$  we shall at most take into account the higher derivatives of  $\phi(y)$ , like  $\phi_{yy}(y)$ ,  $\phi_{yyy}(y)$  instead of  $\phi_y(y)$  and escape from the prescribing of  $a_2$ .

Our numerical calculations confirm that the mean streamwise velocity distribution (averaged over time) due to the induced motion by the wavy elastic vacuum-matter interface in the case of free (vacuum) pumping is dominated by  $R_0$  (or  $\text{Kn}$ ) and the parabolic distribution  $-\Pi_0(1 - y^2)$ .  $R_0$  which defines the boundary value of  $D_y$  has its origin in the  $y$ -gradient of the first-order streamwise velocity distribution, as can be seen in (17).

In addition to the terms mentioned above, there is a perturbation term which varies across the channel:  $\mathcal{C}(y) - \mathcal{C}(1)$ . Let us define it to be

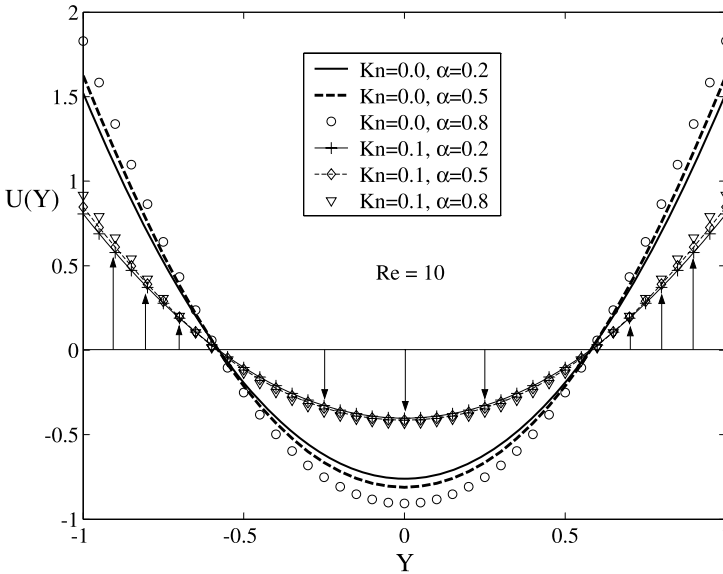
$$F(y) = \frac{-200}{\alpha^2 Re^2} [\mathcal{C}(y) - \mathcal{C}(1)]. \tag{24}$$

We remind the readers that the Reynolds number here is based on the wave speed. The physical trend herein is also the same as those reported in [13, 14] for the slip-flow effects. The slip produces decoupling with the inertia of the wavy interface.

Now, let us define a critical reflux condition as one for which the mean velocity  $U(Y)$  equals to zero at the center-line  $Y = 0$  (cf. Fig. 2). With equations (15), (23)–(24), we have

$$\Pi_{0cr} = Re \overline{\left( \frac{\partial p}{\partial x} \right)}_2 = \frac{[\alpha^2 Re^2 F(0)/200 + \text{Kn} C'(1) - R_0]}{-(1 + 2\text{Kn})} \tag{25}$$





**Fig. 2** Demonstration of the dark states: the mean velocity field  $U(Y)$  for wave numbers  $\alpha = 0.2, 0.5, 0.8$ . The Reynolds number is 10.  $Kn$  is the rarefaction measure (the mean free path of the particles divided by the characteristic length). The arrows are schematic and illustrate the directions of positive and negative  $U(Y)$ . The integration of  $U(Y)$  w.r.t.  $Y$  for these velocity fields gives zero volume (mass) flow rate

**Table 1** Dark states values ( $\Pi_0$ ) for a flat vacuum-matter boundary

Kn	$\alpha$	$Re$			
		0.1	1	10	100
0	0.2	4.5269	4.5269	4.5231	4.3275
	0.5	4.6586	4.6584	4.6359	4.2682
	0.8	4.9238	4.9234	4.8708	4.4488
0.1	0.2	2.4003	2.4000	2.3774	1.2217
	0.5	2.4149	2.4132	2.2731	-0.9054
	0.8	2.4422	2.4379	2.0718	-3.4151

which means the critical reflux condition is reached when  $\Pi_0$  has above value. Pumping against a positive (unit) body forcing greater than the critical value would result in a backward transport (reflux) in the central region of the stream. This critical value depends on  $\alpha$ ,  $Re$ , and  $Kn$ . There will be no reflux if the (unit) body forcing or pressure gradient is smaller than this  $\Pi_0$ . Thus, for some  $\Pi_0$  values less than  $\Pi_{0,cr}$ , the matter (flow) will keep moving or evolving forward. On the contrary, parts of the matter (flow) will move or evolve backward if  $\Pi_0 > \Pi_{0,cr}$ . This result could be similar to that in [19] using different approach or qualitatively related to that of [4]: even for very slow growth of  $\Lambda$ , the gravitationally bound systems become unbound while the nongravitationally bound systems remain bound for certain parameters defined in [4] (e.g.,  $\eta$ ).

We present some of the values of  $\Pi_0(\alpha, Re; Kn = 0, 0.1)$  corresponding to dark states which satisfy  $\int_{-1}^1 U(Y)dY = 0$  in Table 1 where the wave number ( $\alpha$ ) has the range between 0.20 and 0.80; the Reynolds number ( $Re$ ) = 0.1, 1, 10, 100. Here, the dark state is directly

related to the zero-mass-flux (in average) state in bulk sense although there are locally velocity differences across the (confined) space. It means the gravity effects are not absent locally considering the velocity distribution across the confined space. To be specific, the dark state is difficult to be observed since there is no mass flux even though this state happens!

We observe that, from Table 1, as Kn increases from zero to 0.1, the critical  $\Pi_0$  or time-averaged (unit) body forcing decreases significantly. For the same Kn, once Re is larger than 10, critical reflux values  $\Pi_0$  drop rapidly and the wave-modulation effect (due to  $\alpha$ ) appears. The latter observation might be interpreted as the strong coupling between the vacuum-matter boundary and the inertia of the streaming matter-flow. The illustration of the velocity fields for those dark states are shown in Fig. 2. There are three wave numbers:  $\alpha = 0.2, 0.5, 0.8$ . The Reynolds number is 10. Both no-slip and slip (Kn = 0.1) cases are presented. The arrows for slip cases are schematic and represent the direction of positive and negative velocity fields.

Some remarks could be made about these dark states (or solitons): the matter or universe being freezed in the time-averaged sense for specific dissipations (in terms of Reynolds number which is the ratio of wave-inertia and viscous effects) and wave numbers (due to the wavy vacuum-interface or vacuum fluctuations) for either no-slip and slip cases. This particular result might also be related to a changing cosmological term (growing or decaying slowly) or the critical density mentioned in [2]. If we treat the (unit) body forcing as the pressure gradient, then for the same transport direction (say, positive x-direction), the negative pressure (either downstream or upstream) will, at least, occur once the time-averaged flow (the maximum speed of the matter (gas) appears at the center-line) is moving forward!

Meanwhile, the time-averaged transport induced by the wavy interface is proportional to the square of the amplitude ratio (although the small amplitude waves being presumed), as can be seen in (12) or (20), which is qualitatively the same as that presented in [9] for analogous interfacial problems. In brief summary, the entrained transport (pattern, either positive or negative and there being possible dark states) due to the wavy vacuum-matter boundary is mainly tuned by the (unit) body forcing or  $\Pi_0$  for fixed Re (viscous dissipation). Meanwhile,  $\Pi_{0,cr}$  depends strongly on the Knudsen number (Kn, a rarefaction measure) instead of Re or  $\alpha$  (wave number). We hope that in the future we can investigate other issues [25–31] using the present or more advanced approach.

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